Appendix II
Coefficients of the Transmission-Line Equation in the Ferrite Loaded Region

$$a_m^{(11)} = \frac{\omega\left(\mu_0\beta^2 + \mu_\perp \gamma_m^2\right)}{K_m^2} - \frac{K_m^2}{\omega\epsilon}$$

$$a_m^{(12)} = -a_m^{(21)} = -j\beta\gamma_m\omega\left(\mu_0 - \mu_\perp\right)/K_m^2$$

$$a_m^{(22)} = \omega\left(\mu_0\gamma_m^2 + \mu_\perp\beta^2\right)/K_m^2$$

$$b_m^{(12)} = d_m^{(21)} = -j\frac{K}{\mu}\gamma_m$$

$$b_m^{(22)} = -d_m^{(22)} = -\frac{K}{\mu}\beta, \qquad b_m^{(l l)} = d_m^{(l l)} = 0$$

$$c_m^{(11)} = \omega\epsilon, \qquad c_m^{(12)} = c_m^{(21)} = 0$$

$$c_m^{(22)} = \omega\epsilon - \frac{K_m^2}{\omega\mu}$$

where

$$\mu_\perp = \mu - \frac{K^2}{\mu}.$$

Appendix III
Elements of the Determinant

$$F_{nn'}^{(xx)} = \sum_{m=1}^{\infty} \frac{1}{AK_m^2}\left[\beta_0^2\left(Y_{11}^+ - Y_{11}^-\right) - j\beta\gamma_m\left(Y_{12}^- - Y_{21}^-\right) + \gamma_m\left(Y_{22}^+ - Y_{22}^-\right)\right]\tilde{\xi}_n\tilde{\xi}_{n'}$$

$$F_{nn'}^{(xy)} = \sum_{m=1}^{\infty} \frac{1}{AK_m^2}\left[\beta\gamma_m\left(-Y_{11}^+ + Y_{11}^- + Y_{22}^+ - Y_{22}^-\right) - j\beta^2 Y_{12}^- + j\gamma_m^2 Y_{21}^-\right]\tilde{\xi}_n\tilde{\eta}_{n'}$$

$$F_{nn'}^{(yx)} = \sum_{m=1}^{\infty} \frac{1}{Ak_m^2}\left[\beta\gamma_m\left(-Y_{11}^+ + Y_{11}^+ + Y_{22}^+ - Y_{22}^-\right) + j\gamma_m Y_{12}^- + j\beta^2 Y_{21}^-\right]\tilde{\eta}_n\tilde{\xi}_{n'}$$

$$F_{nn'}^{(yy)} = \sum_{m=0}^{\infty} \frac{\epsilon_m}{2AK_m^2}\left[\gamma_m^2\left(Y_{11}^+ - Y_{11}^-\right) + j\beta\gamma_m\left(Y_{12}^- - Y_{21}^-\right) + \beta^2\left(Y_{22}^+\right)\right]\tilde{\eta}_n\tilde{\eta}_{n'} \quad \text{(A1)}$$

where

$$Y_{ll'}^{\pm} = Y_m^{(ll')}(\pm 0)$$

$$\tilde{\xi}_n = \int_{-w}^{w} \sin\gamma_m(x+a)\,\xi_n(x)\,dx$$

$$\tilde{\eta}_n = \int_{-w}^{w} \cos\gamma_m(x+a)\,\eta_n(x)\,dx. \quad \text{(A2)}$$

Note that

$$Y_{12}^+ = Y_{21}^+ = 0.$$

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Propagation in Coupled Unilateral and Bilateral Finlines

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Abstract—**The propagation characteristics of coupled unilateral and bilateral finlines in the even and odd modes are evaluated with the hybrid-mode formulation in the spectral domain. The frequency-dependent guide wavelength is obtained from the solution of the characteristic equation. The characteristic impedance based on the power–voltage definition is also evaluated. Numerical results are presented to study the influence of various structural parameters on the characteristics of finlines.**

I. Introduction

Among various forms of transmission media, finlines have demonstrated potential for their use in millimeter-wave integrated circuits [1], [2]. In a typical finline configuration, a planar circuit is placed in the *E*-plane of a rectangular waveguide. This combines the advantages of both planar circuits and waveguides. Its wide single-mode bandwidth, low dispersion, moderate attenuation and compatibility with semiconductor devices are attractive features at millimeter wavelengths.

The propagation in finlines has been a subject of considerable interest recently. An accurate description of the dispersion can be obtained with the hybrid-mode formulations [3]–[5]. This was first presented by Hofmann [6] who used the space-domain formulation. Subsequently, the spectral-domain formulation was utilized by Knorr and Shayda [7], Schmidt and Itoh [8], as well as by the authors [9]–[12], to determine the propagation characteristics of unilateral and bilateral finlines.

With the increased interest in finlines, there is a need to

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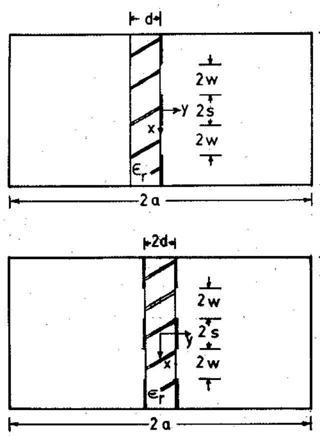

Fig. 1. Coupled unilateral and bilateral finlines.

analyze coupled unilateral and bilateral finlines which are useful in the realization of various millimeter-wave components such as directional couplers and filters. An approximate solution for the propagation in the coupled unilateral finline, presented by Simons and Arora [13], is based on Cohn's procedure for the slot-line on a dielectric substrate. This procedure must take into account a large number of orthogonal modes to achieve reasonable accuracy. Schmidt, Itoh, and Hofmann [14] have recently proposed a full-wave analysis for the symmetric and asymmetric coupled unilateral finlines in the spectral domain.

In this paper, the propagation characteristics of the symmetric coupled unilateral and bilateral finlines [11] are investigated in detail. The full-wave formulation of the spectral-domain technique is utilized to evaluate the guide wavelength. The characteristic impedance is computed on the basis of the power–voltage definition.

II. Analysis

The coupled unilateral and bilateral finline configurations, as shown in Fig. 1, comprise a substrate of relative permittivity ϵ_r in the E-plane of a rectangular waveguide with dimensions $2a$ and $2b$. The thickness of the substrate is d and $2d$ for the coupled unilateral and bilateral finlines, respectively. The slots of width $2w$ are separated by a distance $2s$.

In the spectral-domain analysis of the structure, the Fourier transform of the Green's functions are related to the transforms of the current densities on the conductors and the electric fields in the region of the interface complementary to the conductors, via the equation

$$\begin{bmatrix} \tilde{H}_{11}(\alpha_n,\beta,k_0) & \tilde{H}_{12}(\alpha_n,\beta,k_0) \\ \tilde{H}_{21}(\alpha_n,\beta,k_0) & \tilde{H}_{22}(\alpha_n,\beta,k_0) \end{bmatrix} \begin{bmatrix} \tilde{E}_x(\alpha_n) \\ \tilde{E}_z(\alpha_n) \end{bmatrix} = \begin{bmatrix} \tilde{J}_x(\alpha_n) \\ \tilde{J}_z(\alpha_n) \end{bmatrix} \tag{1}$$

where α_n is the Fourier transform variable, β is the propagation constant, and k_0 is the free-space wavenumber. $\tilde{E}_x$, $\tilde{E}_z$, $\tilde{J}_x$, and $\tilde{J}_z$ are the electric fields in the aperture and the current densities on the conductors, respectively. With the application of Galerkin's procedure and Parseval's theorem, we obtain a set of algebraic equations in terms of the unknown constants of the basis functions. At a given frequency, a nontrivial solution for the propagation constant β is obtained by setting the determinant of the coefficient matrix equal to zero and finding the root of the equation. The guide wavelength is then obtained from the propagation constant.

The characteristic impedance can be evaluated in the spectral domain with the definition

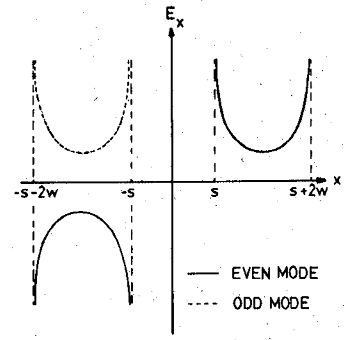

Fig. 2. Variation of the transverse electric field in the even and odd modes.

$$Z_0 = \frac{V_0^2}{2P_{\text{avg}}} \tag{2}$$

where V_0 is the voltage between the fins along the air–dielectric interface and is given by

$$V_0 = \int_s^{s+2w} E_x(z,d)\,dx. \tag{3}$$

The time-averaged power P_{avg} is evaluated as follows:

$$P_{\text{avg}} = 1/2\,\text{Re}\int_{-a}^{a}\int_{-b}^{b}\left(E_x H_y^* - E_y H_x^*\right)dx\,dy. \tag{4}$$

III. Basis Functions

Following the mathematical formulation presented above, the longitudinal and transverse electric fields are expressed in terms of known basis functions with unknown coefficients. The electric fields are appropriately chosen as follows:

$$E_x = \begin{cases} (w^2 - x^2)^{-1/2}, & s < |x| < s+2w \\ 0, & \text{elsewhere} \end{cases} \tag{5}$$

$$E_z = \begin{cases} x(w^2 - x^2)^{1/2}, & s < |x| < s+2w \\ 0, & \text{elsewhere.} \end{cases} \tag{6}$$

The Fourier transform of these functions can be easily evaluated with the definition of the discrete Fourier transform.

The propagation in a coupled finline is described in terms of two independent modes of excitation, known as even and odd modes, which corresponds to even E_z (or odd E_x) and odd E_z (or even E_x), respectively. The variation of the transverse electric field E_x in the even and odd modes is shown in Fig. 2.

IV. Numerical Results

We have developed computer programs which evaluate the guide wavelength and the characteristic impedance at a given frequency of the unilateral and bilateral finlines. We have verified these computer programs by comparing the results with those already reported by Knorr and Shayda [7] for the unilateral finline, and Schmidt and Itoh [8] for the bilateral finline. Since there was good agreement with them for various structural parameters, the numerical results presented in this paper are fairly accurate. Both the longitudinal and transverse electric fields have been taken into account to obtain these results.

The guide wavelength and characteristic impedance have been evaluated for various structural parameters of the finlines on substrates of relative permittivity $\epsilon_r = 2.22$ (RT-Duroid). The present analysis is based on the symmetries of the waveguide structures, and therefore the propagating modes correspond to the electric and magnetic wall symmetry at $x = 0$. In the case of a

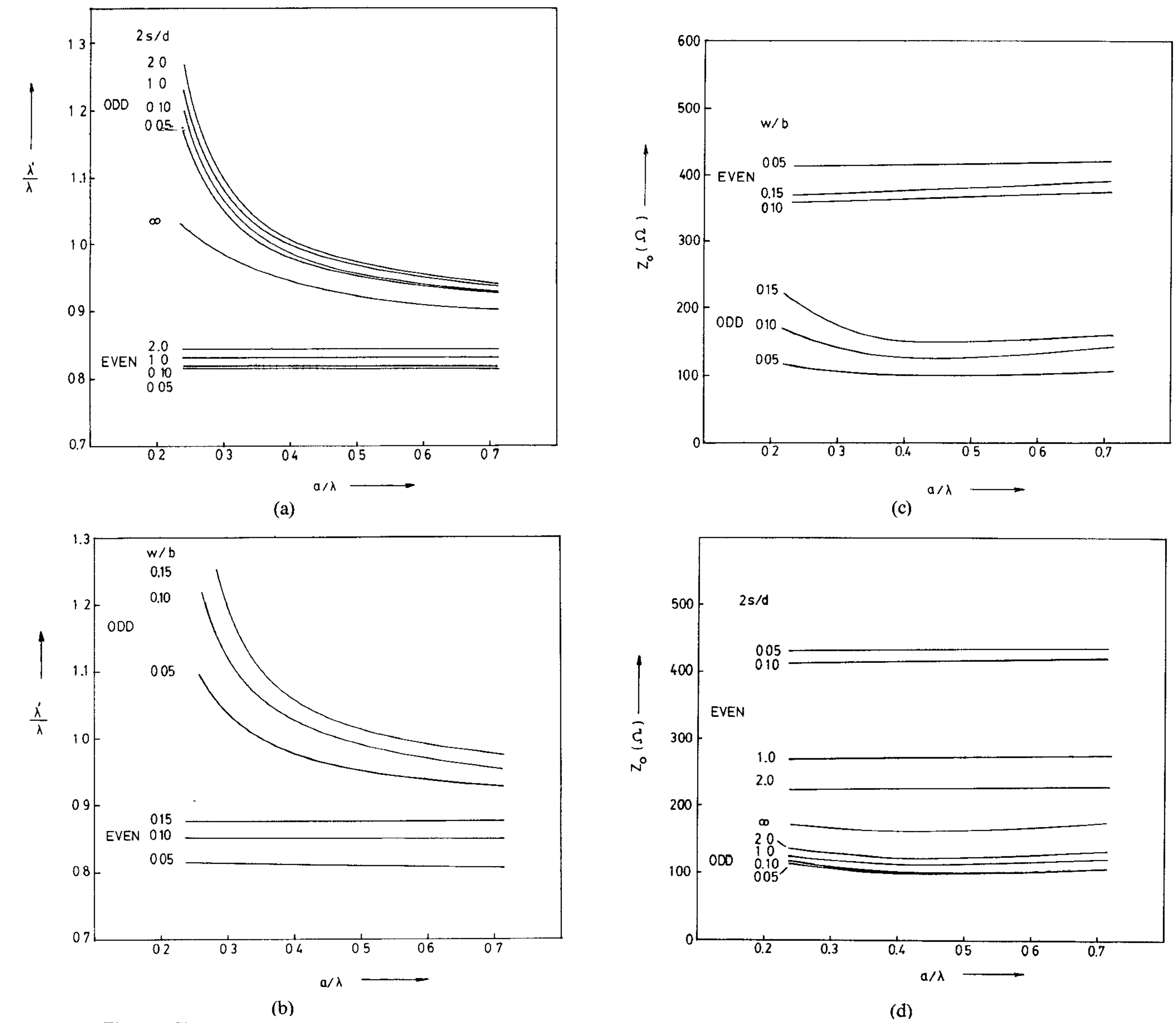

Fig. 3. Characteristics of coupled unilateral finlines in the even and odd modes as a function of normalized frequency a/λ. Normalized guide wavelength (a) with $w/b = 0.05$ and $2s/d = 0.05, 0.1, 1.0, 2.0$; (b) with $2s/d = 0.1$ and $w/b = 0.05, 0.1, 0.15$. Characteristic impedance (c) with $w/b = 0.05$ and $2s/d = 0.05, 0.1, 1.0, 2.0$; (d) with $2s/d = 0.1$ and $w/b = 0.05, 0.1, 0.15$. $d/a = 0.03515$, $a = 0.3556$ cm, $b = 0.1778$ cm, and $\epsilon_r = 2.22$.

coupled bilateral finline, there is an additional magnetic wall symmetry at $y = 0$.

The numerical results for the normalized guide wavelength (λ'/λ) and characteristic impedance (Z_0) in the even and odd modes have been evaluated as a function of the normalized frequency (a/λ). For the coupled unilateral finline, the quantities λ'/λ and Z_0 are plotted in Fig. 3 as a function of normalized frequency (a/λ) for values of normalized slot separation $(2s/d)$ and normalized slot width (w/b). Fig. 3(a) and (b) shows the variation in λ'/λ for various values of $2s/d$ and w/b, respectively. For a fixed value of $2s/d$, we observe that λ'/λ increases slightly in the even mode, whereas in the odd mode it decreases as a function of a/λ. When $2s/d$ is increased for a fixed value of a/λ, it shows an increase in the even mode, but in the odd mode it initially increases and then decreases. Both in the even and odd modes, λ'/λ increases with an increase in w/b.

The computed values of the characteristic impedance in the even and odd modes are shown in Fig. 3(c) and (d). Increasing $2s/d$ at a fixed value of a/λ decreases its value in the even mode and increases in the odd mode. The characteristic impedance in the odd mode for small values of $2s/d$ is also observed to be close to one half of the characteristic impedance of the corresponding unilateral finline with twice the slot width. With an increase in w/b, Z_0 increases in the odd mode, while in the even mode it first decreases and then increases.

For the coupled bilateral finline, the behavior of λ'/λ and Z_0 as a function of a/λ is illustrated in Fig. 4(a)–(d) for various structural parameters s/d and w/b. The variation in λ'/λ, as shown in Fig. 4(a), is similar to that observed for coupled unilateral finline. The propagating mode in the odd mode appears to behave similarly to the one on coplanar striplines. The guide wavelength in the even and odd modes increases with w/b for a fixed value of a/λ (Fig. 4(b)). With an increase in s/d at a fixed value of a/λ, the characteristic impedance, as shown in Fig. 4(c), increases in the even mode, and in the odd mode it initially increases and then decreases. However, as s/d is further

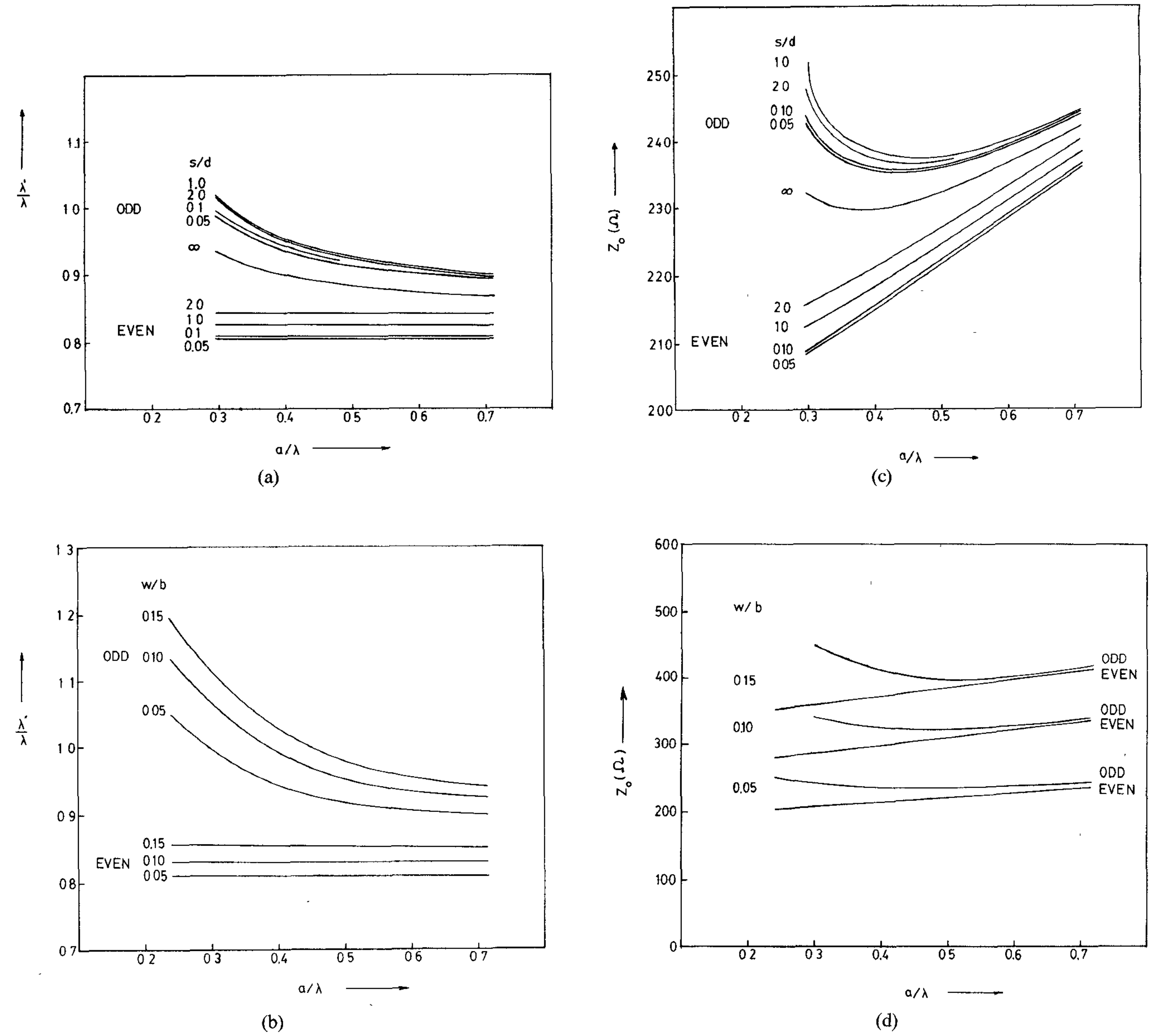

Fig. 4. Characteristics of coupled bilateral finlines in the even and odd modes as a function of normalized frequency a/λ Normalized guide wavelength (a) with $w/b = 0.05$ and $s/d = 0.05, 0.1, 1.0, 2.0$; (b) with $s/d = 0.1$ and $w/b = 0.05, 0.1, 0.15$. Characteristic impedance (c) with $w/b = 0.05$ and $s/d = 0.05, 0.1, 1.0, 2.0$; (d) with $s/d = 0.1$ and $w/b = 0.05, 0.1, 0\ 15$. $d/a = 0.03515$, $a = 0.3556$ cm, $b = 0.1778$ cm, and $\epsilon_r = 2.22$.

increased, both the even- and odd-mode characteristic impedances approach to the characteristic impedance of the corresponding bilateral finline with the same slot width. This is also observed for various values of w/b (Fig. 4(d)). The coupled bilateral finline also supports quasi-TEM propagating modes which correspond to an electric wall symmetry at $y = 0$. These modes have not been considered in this investigation.

V. Conclusion

In this paper, we have presented a detailed description of the propagation in coupled unilateral and bilateral finlines using the hybrid-mode formulation in the spectral domain. The results for the normalized guide wavelength and characteristic impedance have been presented as a function of normalized frequency for some structural parameters. This technique can be utilized to obtain accurate propagation characteristics of any finline geometry.

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Letters

Comments on "Integration Method of Measuring Q of the Microwave Resonators"

P. L. OVERFELT AND D. J. WHITE

The paper of this title[1] suggested integrating the power

$$P(\omega)=P_0\left[1+Q_L^2(\omega/\omega_0-\omega_0/\omega)^2\right]^{-1} \tag{1}$$

transmitted through a resonator [2] across some frequency band in the vicinity of resonance in order to determine the loaded $Q(Q_L)$; it pointed out some advantages over the common approach using the one-half power frequencies. The integrated power was found [1] to be

$$I=\int_{\omega_1}^{\omega_2}P(\omega)\,d\omega=P_0\omega_0Q_L^{-1}\tan^{-1}\left(Q_L\omega_s\omega_0^{-1}\right) \tag{2}$$

where $\omega_s=\omega_2-\omega_1$.

Since we have experienced the usual frustrations in Q-measurement, a new approach is attractive, the first step of which is to check the derivations. This turned out to be more difficult than anticipated, and initially we were unable to come up with a reasonable derivation of (2). However, (1) can be integrated exactly by writing the integral in (2) in the form [3]

$$I/P_0\omega_0=(g/h)\int_{x_1}^{x_2}(cx^2+g)^{-1}\,dx-(f/h)\int_{x_1}^{x_2}(cx^2+f)^{-1}\,dx \tag{3}$$

where $x=\omega/\omega_0$ and c, g, f, and h are appropriate constants. When this is done, we find

$$I=\frac{P_0\omega_0}{2jQ_L\sqrt{(4Q_L^2-1)}}\cdot\left[\left(1+j\sqrt{4Q_L^2-1}\right)\tan^{-1}\frac{\omega\left(1-j\sqrt{4Q_L^2-1}\right)}{2Q_L\omega_0}-\left(1-j\sqrt{4Q_L^2-1}\right)\tan^{-1}\frac{\omega\left(1+j\sqrt{4Q_L^2-1}\right)}{2Q_L\omega_0}\right]_{\omega_1}^{\omega_2} \tag{4}$$

where we have used the relation

$$\left(1-2Q_L^2\pm\sqrt{1-4Q_L^2}\right)^{1/2}=\left(1\pm j\sqrt{4Q_L^2-1}\right)/\sqrt{2}. \tag{5}$$

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